Spezielle Ableitungen

Kehrwertfunktion:

$$f(x)=\frac{1}{x}\quad\left(=x^{-1}\right)\qquad\Rightarrow\qquad f'(x)=-x^{-2}=-\frac{1}{x^2}$$

$$f(x)=\frac{c}{x^n}\qquad\Rightarrow\qquad f'(x)=-n\frac{c}{x^{n+1}}$$

Beispiel:

$$f(x)=\frac{3}{2x^5}=\frac{3}{2}x^{-5}\qquad\Rightarrow\qquad f'(x)=-5\cdot\frac{3}{2}x^{-6}=-\frac{15}{2x^6}$$

Quadratwurzel:

$$f(x)=\sqrt{x}\quad\left(=x^{\frac{1}{2}}\right)\qquad\Rightarrow\qquad f'(x)=\frac{1}{2\cdot\sqrt{x}}$$

$$f(x)=\sqrt{g(x)}=g(x)^{\frac{1}{2}}\qquad\Rightarrow\qquad f'(x)=\frac{g'(x)}{2\cdot\sqrt{g(x)}}$$

Beispiel:

$$f(x)=\sqrt{x^2+3}\qquad\Rightarrow\qquad f'(x)=\frac{2x}{2\sqrt{x^2+3}}=\frac{x}{\sqrt{x^2+3}}$$

Allg. Wurzelfunktion:

$$f(x)=\sqrt[n]{x}\quad\left(=x^{\frac{1}{n}}\right)\qquad\Rightarrow\qquad f'(x)=\frac{1}{n\cdot\sqrt[n]{x^{n-1}}}$$

$$f(x)=\sqrt[n]{g(x)}\quad\left(=g(x)^{\frac{1}{n}}\right)\qquad\Rightarrow\qquad f'(x)=\frac{g'(x)}{n\cdot\sqrt[n]{g(x)^{n-1}}}$$

Beispiele:

$$f(x)=\sqrt[3]{x}\quad\left(=x^{\frac{1}{3}}\right)\qquad\Rightarrow\qquad f'(x)=\frac{1}{3}x^{\frac{2}{3}-1}=\frac{1}{3}x^{-\frac{2}{3}}=\frac{1}{3\cdot\sqrt[3]{x^2}}$$

$$f(x)=\sqrt[3]{6x^2}=\sqrt[3]{6}\cdot x^{\frac{2}{3}}$$

$$\Rightarrow\qquad f'(x)=\sqrt[3]{6}\cdot\frac{2}{3}\cdot x^{\frac{2}{3}-1}=\sqrt[3]{6}\cdot\frac{2}{3}x^{-\frac{1}{3}}=\frac{2\cdot\sqrt[3]{6}}{3\cdot\sqrt[3]{x}}=\frac{2}{3}\cdot\sqrt[3]{\frac{6}{x}}$$

In-Funktion:

$$f(x)=\ln(x)\qquad\Rightarrow\qquad f'(x)=\frac{1}{x}$$

$$f(x)=\ln(g(x))\qquad\Rightarrow\qquad f'(x)=\frac{g'(x)}{g(x)}$$

Beispiel:

$$f(x)=\ln\left(3x^2\right)\qquad\Rightarrow\qquad f'(x)=\frac{6x}{3x^2}=\frac{2}{x}$$

Logarithmusfunktion:

$$f(x)=\log_a(x)=\frac{\ln(x)}{\ln(a)} \quad \Rightarrow \quad f'(x)=\frac{1}{x\cdot\ln(a)}$$

$$f(x)=\log_a(g(x))=\frac{\ln(g(x))}{\ln(a)} \quad \Rightarrow \quad f'(x)=\frac{g'(x)}{g(x)\cdot\ln(a)}$$

Beispiel: $f(x)=\log_5(2x^3) \quad \Rightarrow \quad f'(x)=\frac{6x^2}{2x^3\cdot\ln(5)}=\frac{3}{x\cdot\ln(5)}$

e-Funktion:

$$f(x)=e^x \quad \Rightarrow \quad f'(x)=e^x$$

$$f(x)=e^{g(x)} \quad \Rightarrow \quad f'(x)=e^{g(x)}\cdot g'(x)$$

Beispiel: $f(x)=e^{x^2-3x} \quad \Rightarrow \quad f'(x)=e^{x^2-3x}\cdot(2x-3)$

Exponentialfunktion:

$$f(x)=a^x \quad \left(=e^{x\cdot\ln(a)}\right) \quad \Rightarrow \quad f'(x)=a^x\cdot\ln(a)$$

$$f(x)=a^{g(x)} \quad \left(=e^{g(x)\cdot\ln(a)}\right) \quad \Rightarrow \quad f'(x)=a^{g(x)}\cdot g'(x)\cdot\ln(a)$$

Beispiel: $f(x)=3^{2x+1} \quad \Rightarrow \quad f'(x)=3^{2x+1}\cdot 2\cdot\ln(3)$

Trigonometr. Fkt.:

$$f(x)=\sin(x) \quad \Rightarrow \quad f'(x)=\cos(x)$$

$$f(x)=\cos(x) \quad \Rightarrow \quad f'(x)=-\sin(x)$$

$$f(x)=\sin(g(x)) \quad \Rightarrow \quad f'(x)=g'(x)\cdot\cos(g(x))$$

$$f(x)=\cos(g(x)) \quad \Rightarrow \quad f'(x)=-g'(x)\cdot\sin(g(x))$$

$$f(x)=\tan(x) \quad \left(=\frac{\sin(x)}{\cos(x)}\right) \quad \Rightarrow \quad f'(x)=\frac{1}{\cos^2(x)}$$

$$f(x)=\cot(x) \quad \left(=\frac{\cos(x)}{\sin(x)}\right) \quad \Rightarrow \quad f'(x)=-\frac{1}{\sin^2(x)}$$

$$f(x)=\tan(g(x)) \quad \Rightarrow \quad f'(x)=\frac{g'(x)}{\cos^2(g(x))}$$

$$f(x)=\cot(g(x)) \quad \Rightarrow \quad f'(x)=-\frac{g'(x)}{\sin^2(g(x))}$$

Beispiele: $f(x)=\sin(3x^2+1) \quad \Rightarrow \quad f'(x)=6x\cdot\cos(3x^2+1)$

$$f(x)=\tan(x^4) \quad \Rightarrow \quad f'(x)=\frac{4x^3}{\cos^2(x)}$$